Statistics

Fall 2022

Lecture 24

Estimating Parameters

SG 22 & 23

What is a parameter?

It is a numerical measurement related to population.

To estimate Parameters, we need to use Corresponding statistic.

What is Statistic?

It is a numerical value related to Sample.

Parameters (Population)		Statistic (Sample)
Population Proportion P	$\Longleftrightarrow$	Sample Proportion $\hat{p}$ (P-hat)
Population Mean μ	$\Longleftrightarrow$	Sample Mean $\bar{x}$ (x-bar)
Population Standard deviation σ	$\Longleftrightarrow$	Sample Standard deviation s

To estimate Population we must use Sample

Proportion	p	Proportion	$\hat{p}$
Mean	μ	Mean	$\bar{x}$
Standard deviation	σ	Standard deviation	s

Point-estimate

Once we estimate parameters, we get range of values, and it is called Confidence Interval

Every Confidence Interval Comes with Confidence level.

$(1-\alpha)\cdot 100\%$

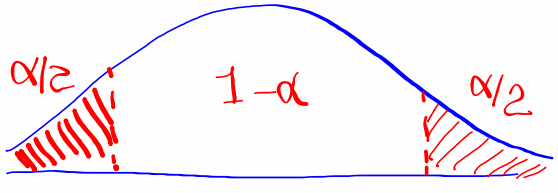
α → Alpha

$0 < \alpha < 1$

Significance level

$1-\alpha$ is the middle area of the graph of Prob. dist.

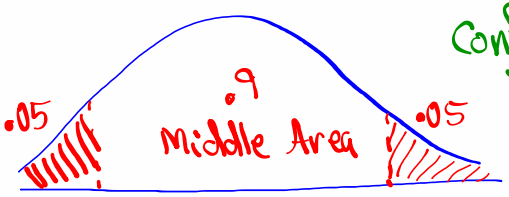
If α not given, we use .05.



If $\alpha = .1$

$(1-\alpha)\cdot 100\% = (1-.1)\cdot 100\% = .9\,(100\%) = 90\%$

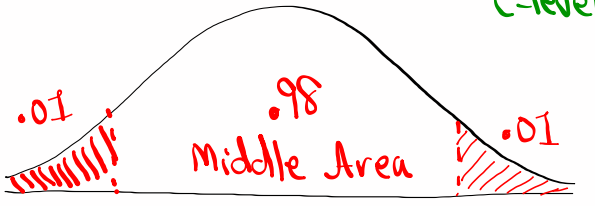
Conf. level
"C-level"



If $\alpha = .02$

$(1-\alpha)\cdot 100\% = (1-.02)\cdot 100\% = (.98)\cdot 100\% = 98\%$

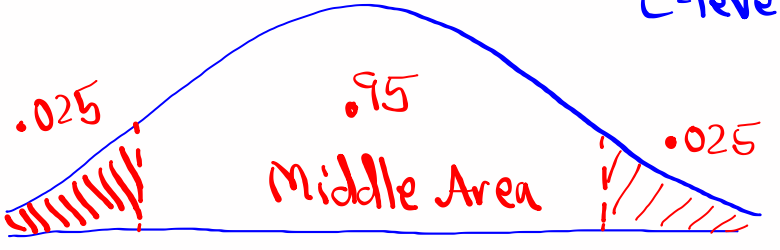
C-level



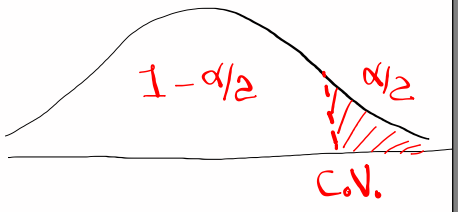
If α not given

Use $\alpha = .05$ → $(1-\alpha)\cdot 100\% = 95\%$

C-level



The value that separates the top $\alpha/2$ area from the rest is called Critical Value.



Ex Find $Z_{\alpha/2}$ for $\alpha = .04$

$\alpha/2 = .04/2 = .02$

$Z_{.02} = \text{invNorm}(.98, 0, 1)$

$= \boxed{2.054}$

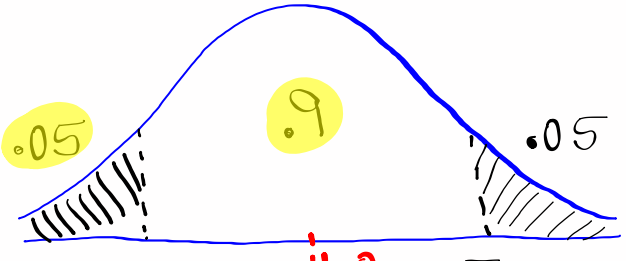
C-level: 96%

Find $Z_{\alpha/2}$ for 90% C-level.

Middle Area

$1 - .9 = .1$

$.1 \div 2 = .05$



$Z_{.05} = \text{invNorm}(.95, 0, 1) = \boxed{1.645}$

Estimating Population Proportion p:

$$\hat{p} - E < p < \hat{p} + E$$

$\hat{p}$: Sample Proportion ("Point-Estimate")

E: Margin of error

$$\hat{p} = \frac{x}{n}$$

x ← # of successes (favorable respo.)

n ← Sample Size

$$\hat{q} = 1 - \hat{p}$$

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$z_{\alpha/2}$ is the Critical Value for $(1-\alpha)\cdot 100\%$ Confidence level.

I Surveyed 250 Students, and 200 had iPhone.

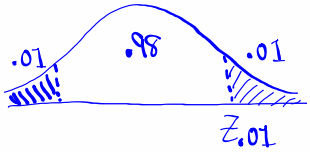
$n = 250$

$x = 200$

$\hat{p} = \frac{x}{n} = \frac{200}{250} = .8$ $\boxed{\hat{p} = .8}$

$\hat{q} = 1 - \hat{p} = 1 - .8 = .2$ $\boxed{\hat{q} = .2}$

I wish to construct 98% Confidence Interval for the prop. of all Students that have iPhone. C-level: .98



$z_{.01} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 2.326 \cdot \sqrt{\frac{(.8)(.2)}{250}}$

$= \boxed{.06}$

$\hat{p} - E < p < \hat{p} + E$

$.8 - .06 < p < .8 + .06$

$\checkmark\ \boxed{.74 < p < .86}$

→ we are 98% confident that between 74% to 86% of all Students have iPhone.

Now using TI:

STAT → TESTS ↓ 1-PropZInt

x: 200

n: 250

C-level: .98

Calculate

.74115 , .85885

$.74 < p < .86$

$E = \frac{.86 - .74}{2} = .06$

I surveyed 150 students (n) and 8% of them ($\hat{p}$) were Smokers.

$n = 150$

$\hat{p} = .08$

$\Rightarrow x = n\hat{p} = 150(.08) = 12$

if decimal → Round-up

find 90% Conf. interval for the prop. of all students that are Smokers.

↳ C-level: .9

Stat TESTS → 1-PropZInt

$x = 12$

$n = 150$

C-level: .9

Calculate

$.044 < p < .116$

$4\% < p < 12\%$

We are 90% Confident that between 4% and 12% of all students Smoke.

$E = \frac{.116 - .044}{2} = .036$

$E \approx 4\%$

Given $.286 < p < .442$

Sample Prop. $\hat{p} = \frac{.442 + .286}{2}$ Point-Estimate

$= .364$

Margin of error $E = \frac{.442 - .286}{2}$

$= .078 \approx 8\%$

$29\% < p < 44\%$

I surveyed 400 voters, and 22% of them planned to support certain candidate on the upcoming election.

$n = 400$

$\hat{p} = .22$ $\Rightarrow x = n\hat{p} = 400(.22) = 88$

if decimal $\Rightarrow$ Round-up

Find Confidence interval for the prop. of all voters in support of that candidate

No C-level $\Rightarrow$ use 95%

1-PropZInt

x: 88

n: 400

C-level: .95

$.179 < p < .261$

$E = \frac{.261 - .179}{2} = .041$

$E \approx 4\%$

$18\% < p < 26\%$

How to determine the minimum Sample Size when working with pop. proportion:

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

with some algebra $\Rightarrow$ $$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$$

if decimal $\Rightarrow$ Round-up

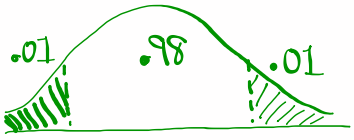
if $\hat{p}$ & $\hat{q}$ are both unknown

use .5 for both $\Rightarrow$ $$n = .25\left(\frac{z_{\alpha/2}}{E}\right)^2$$

Find minimum Sample Size needed to Construct 98% Conf. interval for the prop. of all students that have iPhone. Assume $\hat{p} = .75$.

have $E = 4\%$

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2 = (.75)(.25)\left(\frac{2.326}{.04}\right)^2$$



$n = 634.017$

$n = 635$

$z_{\alpha/2} = \text{invNorm}(.99, 0, 1) = 2.326$

If $\hat{p}$ & $\hat{q}$ were both unknown

$$n = .25\left(\frac{2.326}{.04}\right)^2 \Rightarrow n \approx 846$$

Find minimum Sample Size needed to Construct 90% Conf. interval for pop. prop. with margin of error not to exceed 5%.

a) Assume $\hat{p} = .6$

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$$

$$= (.6)(.4)\left(\frac{1.645}{.05}\right)^2$$

$$n \approx 260$$

$z_{\alpha/2} = \text{invNorm}(.95, 0, 1) = 1.645$

b) Assume $\hat{p}$ & $\hat{q}$ are both unknown.

$$n = .25\left(\frac{z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{1.645}{.05}\right)^2$$

$$n \approx 271$$

1) Work on SG 22
2) Watch the videos on the right side of SG 22 & SG 23, and make notes.
3) We finish SG 23 tomorrow.
4) Final Exam: one week from this Thursday.